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STEADY-STATE AVAILABILITY AND PROBABILITIES OF M/G/1 SYSTEM WITH IMPERFECT COVERAGE PROBABILITY

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Abstract

One of the most difficult factors in the effective installation of a service system is the availability of a continuous power source. We examine a fault-tolerant power generating system of finite operational units with warm standby unit provisioning in this work. Each operational and standby unit's time-to-failure is considered to be exponentially distributed. The time-to-repair for the failing units by the single service facility follows the arbitrary distribution. For modeling purpose, we also included real operating behaviors such as imperfect coverage of unit failure, switching failure of standby unit, reboot delay, switch over delay, and so on. The one and only input required for the assessment of the explicit equation for the system's steady-state probability is the repair time distribution's Laplace-Stieltjes transform (LST). The numerical findings for the following repair time distributions are presented: exponential (M), n-stage Erlang (Er_n), and deterministic (D). Concluding evaluations are also provided.

Keywords: *Imperfect coverage, Warm standby, Laplace-Stieltjes transformation, Repair time distributions, Switch over delay*

1. **Introduction:** The random failures and systematic repairs of machining system have a major influence on machining system's output and productivity. As a result, effective maintenance and repair plans are essential to ensure that the machining system runs continuously and smoothly without interruption. The influence of a random failure of the system on the efficiency of ongoing manufacturing must also be considered while constructing and investigating the system. Gupta and Rao (1994) established a recursive method to compute the steady state probabilities of the machine interference model: (M/G/1) K. Moustafa (1997) worked on reliability analysis of K-out-of-N: G systems with dependent failures and imperfect coverage. Huang and Ke (2009) investigate

comparative analysis on a redundant repairable system with different configurations. Wang et al. (2014) optimized an M/G/1 machine repair problem with multiple imperfect coverage. Liou (2015) worked on analysis of the machine repair problem with multiple vacations and working breakdowns. Shekhar et al. (2017) investigated a survey on queues in machining system: progress from 2010 to 2017. Shekhar et al. (2019) considered a fault-tolerant redundant repairable system with different failures and delays. Yang and Tsao (2019) obtained reliability and availability analysis of standby systems with working vacations and retrial of failed components. Shekhar et al. (2020) discussed M/G/1 fault-tolerant machining system with imperfection. Chen and Wang (2020) analysed reliability and sensitivity of a retrial machine repair problem with warm standbys and imperfect coverage. Breneman et al. (2022) Sanusi and Yusuf (2023) worked on availability and cost–benefit analysis of a fault tolerant series–parallel system with human-robotic operators.

The analysis of this research paper is structured as follows: we give introduction in section 1. Section 2 has a full model explanation that includes notations also. Section 3 introduces the recursive approach for computing the steady-state probability and availability of the repairable system. In section 4, various forms of repair time distributions are employed, and explicit equations for state probability and availability are constructed. Section 5 explored the sensitivity of parameters. Section 6 draws conclusions and discussions.

2. Model Explanation:

The system under consideration is applicable to a wide range of electricity and electronics industries where system availability is essential and repairs are required. For this purpose of the modeling, we assume that the system has one primary unit and one standby unit to provide a consistent and uninterrupted power supply to the service system. The system has been constructed with the assumption that the timeframes to failure and repair of units (main and warm standby) are distributed exponentially. The failure rates for the primary and warm standby units are λ and α $(0 < \alpha < \lambda < 1)$ respectively. To identify failure or provide immediate repair, all primary and standby units are under the care of a single repairman and automated monitoring device. The automatic monitoring device identifies the issue with the performance of the system with a high degree of certainty. After successful coverage, the failing unit is quickly replaced by an available standby unit with an exponentially distributed switchover time of $1/\sigma$. The switchover may fail with failure probability p.

When a units' failure is not successfully covered, the system enters an unsafe failure state, and the failed unit is removed from the system through a reboot procedure. There are two types of reboot delay rate β_1 (for standby unit) and β_2 (for standby unit). The delay between reboots is considered to be exponentially distributed. Because the reboot procedure is so quick, the probability of any other event occurring is quite unlikely. In an automated monitoring device, for continuous system operation, the available standby unit promptly changes in place of the failed running unit, with a switching failure probability of q. The failed unit is repairable and is immediately delivered to a single repairman on a first come, first served (FCFS) basis. Repair times are identically and independently dispersed random variables (iidrvs) with a probability density function b(u), distribution function B(u), and mean repair time by b_1. A unit that has been repaired is as good as new.

Notations: M (t): Number of primary component in the system at time t, (initially)

N (t): Number of warm standby component in the system

U (t): Lasting repair time for the component being repaired

λ: Failure rate of primary component in the system

α: Failure rate of a warm standby component in the system

c: Coverage probability (Detection rate)

σ: Switchover time

β_1: Reboot delay rate for standby component

β_2: Reboot delay rate for primary component

b (u): Probability density functions for repair distribution

b_1: Mean repair time

$P_{m,n}$ (t): Probability that at time t, where m and n are operating and warm standby units respectively. Where m, n = 0, 1

$P^*_{m,n}(s)$: Laplace-Stieltjes transformation of $P_{m,n}$ (t),

$P^{*(1)}_{m,n}(s)$: First order derivative of $P_{m,n}$ (t) with respect to s

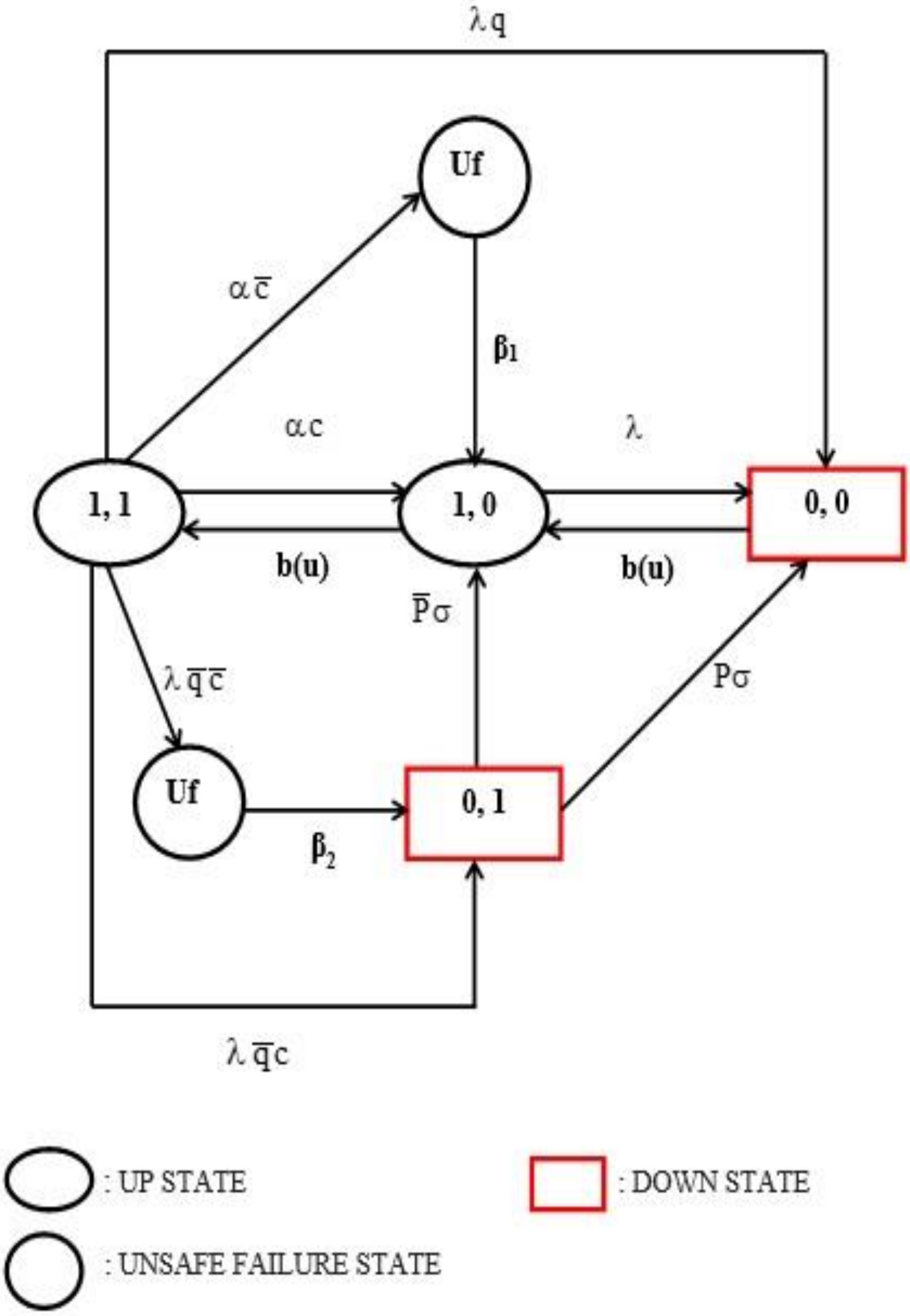

Fig. 1 STATE TRANSITION DIAGRAM

3. Steady-state probabilities and availability:

We define the following state probabilities for $u \geq 0, t \geq 0$

$$P_{1,1}(u,t) = \mathrm{Prob}\{M(t)=1, N(t)=1, I(t)=0, u \leq U(t) \leq u+du\}$$

$$P_{1,0}(u,t) = \mathrm{Prob}\{M(t)=1, N(t)=0, I(t)=0, u \leq U(t) \leq u+du\}$$

$$P_{0,0}(u,t) = \mathrm{Prob}\{M(t)=0, N(t)=0, I(t)=0, u \leq U(t) \leq u+du\}$$

$$Q_{0,1}(u,t) = \Pr ob\{M(t)=0, N(t)=1, I(t)=1, u \le U(t) \le u+du\}$$

$$R_O(u,t) = \Pr ob\left\{\begin{matrix}\text{imperfect coverage of the failed standby machine,} \\ I(t)=1, u \le U(t) \le u+du\end{matrix}\right\}$$

$$R_S(u,t) = \Pr ob\left\{\begin{matrix}\text{imperfect coverage of the failed standby machine,} \\ I(t)=1, u \le U(t) \le u+du\end{matrix}\right\}$$

Therefore,

$$P_{1,1}(t) = \int_0^\infty P_{1,1}(u,t)du\,; \quad P_{1,0}(t) = \int_0^\infty P_{1,0}(u,t)du\,; \quad P_{0,0}(t) = \int_0^\infty P_{0,0}(u,t)du$$

$$Q_{0,1}(t) = \int_0^\infty Q_{0,1}(u,t)du\,; \quad R_O(t) = \int_0^\infty R_O(u,t)du\,; \quad R_S(t) = \int_0^\infty R_S(u,t)du$$

We have the following differential difference equations of each state.

$$\frac{d}{dt}P_{1,1}(t) = -(\lambda+\alpha)P_{1,1}(t) + P_{1,0}(0,t) \tag{1}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)P_{1,0}(u,t) = -\lambda P_{1,0}(u,t) + \alpha c P_{1,1}(u,t) + b(u)P_{0,0}(0,t) + \beta_1 R_S(u,t) + (1-p)\sigma Q_{0,1}(u,t) \tag{2}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)P_{0,0}(u,t) = \lambda P_{1,0}(u,t) + \lambda q P_{1,1}(u,t) + P\sigma Q_{0,1}(u,t) \tag{3}$$

$$\frac{d}{dt}Q_{1,1}(t) = -\sigma Q_{0,1}(t) + \lambda(1-q)cP_{1,1}(t) + \beta_2 R_O(t) \tag{4}$$

$$\frac{d}{dt}R_O(t) = -\beta_2 R_O(t) + \lambda(1-q)(1-c)P_{1,1}(t) \tag{5}$$

$$\frac{d}{dt}R_S(t) = -\beta_1 R_S(t) + \alpha(1-c)P_{1,1}(t) \tag{6}$$

We illustrate the following results in steady state (as $t \to \infty$).

$$P_{1,1} = \lim_{t\to\infty} P_{1,1}(t); \quad P_{1,0} = \lim_{t\to\infty} P_{1,0}(t); \quad P_{0,0} = \lim_{t\to\infty} P_{0,0}(t)$$

$$Q_{0,1} = \lim_{t\to\infty} Q_{0,1}(t); \quad R_O = \lim_{t\to\infty} R_O(t); \quad R_S = \lim_{t\to\infty} R_S(t)$$

and

$$P_{1,1}(u) = \lim_{t\to\infty} P_{1,1}(u,t); \quad P_{1,0}(u) = \lim_{t\to\infty} P_{1,0}(u,t); \quad P_{0,0}(u) = \lim_{t\to\infty} P_{0,0}(u,t)$$

$$Q_{0,1}(u) = \lim_{t\to\infty} Q_{0,1}(u,t); \quad R_O(u) = \lim_{t\to\infty} R_O(u,t); \quad R_S(u) = \lim_{t\to\infty} R_S(u,t)$$

Further we define,

$$P_{1,1}(u) = b(u)P_{1,1} \tag{7}$$

$$Q_{0,1}(u) = b(u)P_{0,1} \tag{8}$$

$$R_O(u) = b(u)R_O \tag{9}$$

$$R_S(u) = b(u)R_S \tag{10}$$

We get the following steady state equations from (1) - (6), using the equations (7) - (10).

$$0 = -(\lambda + \alpha)P_{1,1} + P_{1,0}(0) \tag{11}$$

$$-\frac{\partial}{\partial u} P_{1,0}(u) = -\lambda P_{1,0}(u) + \alpha c b(u)P_{1,1} + b(u)P_{0,0}(0) + \beta_1 b(u)R_S + (1-p)\sigma b(u)Q_{0,1} \tag{12}$$

$$-\frac{\partial}{\partial u} P_{0,0}(u) = \lambda P_{1,0}(u) + \lambda q b(u)P_{1,1} + P\sigma b(u)Q_{0,1} \tag{13}$$

$$0 = -\sigma Q_{0,1} + \lambda(1-q)cP_{1,1} + \beta_2 R_O \tag{14}$$

$$0 = -\beta_2 R_O + \lambda(1-q)(1-c)P_{1,1} \tag{15}$$

$$0 = -\beta_1 R_S + \alpha(1-c)P_{1,1} \tag{16}$$

We obtain the following results from equations (11), (15) and (16).

$$P_{1,0}(0) = (\lambda + \alpha)P_{1,1} \tag{17}$$

$$R_O = \frac{\lambda(1-q)(1-c)}{\beta_2} P_{1,1} \tag{18}$$

$$R_S = \frac{\alpha(1-c)}{\beta_1} P_{1,1} \tag{19}$$

We have from equation (14) by using equation (18).

$$Q_{0,1} = \frac{\lambda(1-q)}{\sigma} P_{1,1} \tag{20}$$

Here we define Laplace-Stieltjes transform in the term of Laplace variable s for probability density function $b(u)$ of repair times and state probabilities as following.

$$B^*(s) = \int_0^\infty e^{-su}\, dB(u) = \int_0^\infty e^{-su} b(u)du \ , \quad P_{1,0}^*(s) = \int_0^\infty e^{-su} P_{1,0}(u)du$$

$$P_{1,0} = P_{1,0}^*(0) = \int_0^\infty P_{1,0}(u)du \ , \quad \int_0^\infty e^{-su} \frac{d}{du} P_{1,0}(u)du = sP_{1,0}^*(s) - P_{1,0}(0)$$

and

$$P_{0,0}^*(s) = \int_0^\infty e^{-su} P_{0,0}(u)du \ , \quad P_{0,0} = P_{0,0}^*(0) = \int_0^\infty P_{0,0}(u)du \ ,$$

$$\int_0^\infty e^{-su} \frac{d}{du} P_{0,0}(u)du = sP_{0,0}^*(s) - P_{0,0}(0)$$

Now taking Laplace-Stieltjes transform (LST) on both sides of equations (12) and (13), we get

$$(\lambda - s)P^{*}_{1,0}(s) = \{\alpha c P_{1,1} + P_{0,0}(0) + \beta_1 R_S + (1-p)\sigma Q_{0,1}\}B^{*}(s) - P_{1,0}(0) \tag{21}$$

$$-sP^{*}_{0,0}(s) = \lambda P^{*}_{1,0}(s) + \lambda q B^{*}(s)P_{1,1} + P\sigma B^{*}(s)Q_{0,1} - P_{0,0}(0) \tag{22}$$

Setting $s = \lambda$ and substituting the values of equations (17), (19) and (20) into equation (21), we obtain

$$P_{0,0}(0) = \frac{\lambda + \alpha - \{\lambda + \alpha(1-p)(1-q)\}B^{*}(\lambda)}{B^{*}(\lambda)} P_{0,0} \tag{23}$$

Setting $s = 0$ and substituting the values of equations (20) and (23) into equation (22), we get

$$P^{*}_{1,0}(0) = \frac{(\lambda + \alpha)(1 - B^{*}(\lambda))}{\lambda B^{*}(\lambda)} P_{1,1} \tag{24}$$

Now differentiating equation (21) with respect to s and setting $s = 0$ and putting the value of

$b_1 = -B^{*(1)}(0) = -\left(\frac{\partial B^{*}(s)}{\partial s}\right)_{S=0}$ then we obtain the following expression,

$$\lambda P^{*(1)}_{1,0}(0) = P^{*}_{1,0}(0) - b_1\{\alpha c P_{1,1} + P_{0,0}(0) + \beta_1 R_S + (1-p)\sigma Q_{0,1}\} \tag{25}$$

Putting the values of equations (19), (20), (23) and (24) into equation (25), we get

$$P^{*(1)}_{1,0}(0) = \frac{(\lambda + \alpha)(1 - \lambda b_1 - B^{*}(\lambda))}{\lambda^2 B^{*}(\lambda)} P_{1,1} \tag{26}$$

Similarly differentiating equation (22) with respect to s and setting $s = 0$ we obtain

$$P^{*}_{0,0}(0) = -\lambda P^{*(1)}_{1,0}(0) + \lambda q b_1 P_{1,1} + P\sigma b_1 Q_{0,1} \tag{27}$$

We get the resulting value after substituting the equations (20) and (26) into equation (27)

$$P^{*}_{0,0}(0) = \left[\frac{(\lambda + \alpha)\{B^{*}(\lambda) + \lambda b_1 - 1\} + \lambda^2 b_1\{p(1-q) + q\}B^{*}(\lambda)}{\lambda^2 B^{*}(\lambda)}\right] P_{1,1} \tag{28}$$

Following is the normalizing condition to obtain the value of $P_{1,1}$

$$P_{1,1} + P^{*}_{1,0}(0) + P_{0,0}(0) + Q_{0,1} + R_O + R_S = 1 \tag{29}$$

Putting the values of $P^{*}_{1,0}(0), P_{0,0}(0), Q_{0,1}, R_O$ and R_S into equation (29), we obtain the resulting value

$$P_{1,1} = \frac{\sigma\beta_1\beta_2 B^{*}(\lambda)}{M} \tag{30}$$

Where $M = \begin{bmatrix} \sigma\beta_1\beta_2\{1 + \lambda b_1(p(1-q) + q)\} + \lambda\beta_1(1-q)\{\beta_2 + \sigma(1-c)\} \\ + \sigma\beta_2\alpha(1-c) \end{bmatrix} B^{*}(\lambda) + \sigma\beta_1\beta_2 b_1(\lambda + \alpha)$

After getting the explicit expression for the state probability $P_{1,1}$, we get the remaining state probabilities as follow

$$P^*_{1,0}(0) = \frac{\sigma\beta_1\beta_2(\lambda+\alpha)(1-B^*(\lambda))}{\lambda M} \tag{31}$$

$$P^*_{0,0}(0) = \frac{\sigma\beta_1\beta_2\left[(\lambda+\alpha)(B^*(\lambda)+\lambda b_1-1)+\lambda^2 b_1\{p(1-q)+q\}B^*(\lambda)\right]}{\lambda M} \tag{32}$$

$$Q_{0,1} = \frac{\lambda(1-q)\beta_1\beta_2 B^*(\lambda)}{M} \tag{33}$$

$$R_O = \frac{\lambda\sigma\beta_1(1-q)(1-c)B^*(\lambda)}{M} \tag{34}$$

$$R_S = \frac{\alpha\sigma\beta_2(1-c)B^*(\lambda)}{M} \tag{35}$$

Now the explicit expression for the steady state availability can be obtained as following

$$A_V = P_{1,1} + P^*_{1,0}(0) \tag{36}$$

Substituting the values of equations (30) and (31) into equation (36), we get

$$A_V = \frac{\sigma\beta_1\beta_2\{\lambda+\alpha(1-B^*(\lambda))\}}{\lambda M} \tag{37}$$

4. Special Scenarios:

The explicit equation for state probabilities and system availability may be easily determined for different continuous distributions of service durations based on the flow of the solution. We just need a Laplace-Stieltjes transform (LST) of the repair time distribution for the given recursive technique. We offer the explicit expression of state probabilities as well as the system availability for exponential (M), Erlangian with n stages (Er_n) and Deterministic (D) distributions.

a) **Exponential distribution:** The repair times are distributed exponentially with a mean rate. It belongs to the gamma distribution. The Laplace-Sieltjes transform of the probability density function $b(u) = \mu e^{-\mu u}$ is given by

$$B^*(\lambda) = \frac{\mu}{\lambda+\mu}$$

As a result, we have $b_1 = \frac{1}{\mu}$. We have the following expressions:

$$P_{1,1}=\frac{\sigma\beta_1\beta_2\mu^2}{\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)} \tag{38}$$

$$P_{1,0}^{*}(0)=\frac{\sigma\mu\beta_1\beta_2(\lambda+\alpha)}{\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)} \tag{39}$$

$$P_{0,0}^{*}(0)=\frac{\sigma\beta_1\beta_2\left[(\lambda+\alpha)\left(\mu^2+\lambda(\lambda+\mu)-\mu(\lambda+\mu)\right)+\mu\lambda^2\{p(1-q)+q\}\right]}{\lambda\left[\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)\right]} \tag{40}$$

$$Q_{0,1}=\frac{\lambda\mu^2\beta_1\beta_2(1-q)}{\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)} \tag{41}$$

$$R_O=\frac{\lambda\sigma\mu^2\beta_1(1-q)(1-c)}{\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)} \tag{42}$$

$$R_S=\frac{\alpha\sigma\mu^2\beta_2(1-c)}{\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)} \tag{42}$$

$$A_V=\frac{\sigma\beta_1\beta_2\{\lambda+\alpha+\mu\}\mu}{\begin{bmatrix}\sigma\beta_1\beta_2\{\mu+\lambda(p(1-q)+q)\}+\mu\beta_1(1-q)\{\beta_2+\sigma(1-c)\}\\+\sigma\mu\beta_2\alpha(1-c)\end{bmatrix}\mu+\sigma\beta_1\beta_2(\lambda+\mu)(\lambda+\alpha)} \tag{43}$$

b) n- stage Erlangian distribution:

The failure repair time has an n-stage Erlang distribution with shape parameter n and rate μ. It is also a gamma distribution member and the sum of n independent exponential variables with mean $1/\mu$ each, implying that repair is done in n stages with mean repair rate μ. In this particular case, we have

$$B^{*}(\lambda)=\left(\frac{n\mu}{n\mu+\lambda}\right)^{n}$$

We have resulting stepwise findings for the explicit expressions for state probabilities and system availability.

$$P_{1,1} = \frac{\sigma\beta_1\beta_2\left(\frac{n\mu}{n\mu+\lambda}\right)^n}{M_1} \tag{44}$$

Where

$$M_1 = \left[\begin{matrix}\sigma\beta_1\beta_2\left\{1+\frac{\lambda}{\mu}\left(p(1-q)+q\right)\right\}+\lambda\beta_1(1-q)\{\beta_2+\sigma(1-c)\} \\ +\sigma\beta_2\alpha(1-c)\end{matrix}\right]\left(\frac{n\mu}{n\mu+\lambda}\right)^n + \frac{\sigma\beta_1\beta_2(\lambda+\alpha)}{\mu}$$

$$P^*_{1,0}(0) = \frac{\sigma\beta_1\beta_2(\lambda+\alpha)\left(1-\left(\frac{n\mu}{n\mu+\lambda}\right)^n\right)}{\lambda M_1} \tag{45}$$

$$P^*_{0,0}(0) = \frac{\sigma\beta_1\beta_2\left[(\lambda+\alpha)\left(\left(\frac{n\mu}{n\mu+\lambda}\right)^n+\frac{\lambda}{\mu}-1\right)+\frac{\lambda^2}{\mu}\{p(1-q)+q\}\left(\frac{n\mu}{n\mu+\lambda}\right)^n\right]}{\lambda M_1} \tag{46}$$

$$Q_{0,1} = \frac{\lambda(1-q)\beta_1\beta_2\left(\frac{n\mu}{n\mu+\lambda}\right)^n}{M_1} \tag{47}$$

$$R_O = \frac{\lambda\sigma\beta_1(1-q)(1-c)\left(\frac{n\mu}{n\mu+\lambda}\right)^n}{M_1} \tag{48}$$

$$R_S = \frac{\alpha\sigma\beta_2(1-c)\left(\frac{n\mu}{n\mu+\lambda}\right)^n}{M_1} \tag{49}$$

$$A_V = \frac{\sigma\beta_1\beta_2\left\{\lambda+\alpha\left(1-\left(\frac{n\mu}{n\mu+\lambda}\right)^n\right)\right\}}{\lambda M_1} \tag{50}$$

c) Deterministic distribution:

It is also known as a degenerate distribution since it only accepts a single value. The repair time follows a deterministic distribution with p being the mean rate. In this scenario, we have the following Laplace-Sieltjes transform of the probability density function.

$$B^*(\lambda) = e^{-(\lambda/\mu)}$$

Hence, we have $b_1 = \frac{1}{\mu}$. We obtain the following expressions:

$$P_{1,1} = \frac{\sigma\beta_1\beta_2 e^{-(\lambda/\mu)}}{M_2} \tag{51}$$

Where

$$M_2 = \left[\begin{array}{c}\sigma\beta_1\beta_2\left\{1+\frac{\lambda}{\mu}(p(1-q)+q)\right\}+\lambda\beta_1(1-q)\{\beta_2+\sigma(1-c)\} \\ +\sigma\beta_2\alpha(1-c)\end{array}\right]e^{-(\lambda/\mu)} + \frac{\sigma\beta_1\beta_2(\lambda+\alpha)}{\mu}$$

$$P^*_{1,0}(0) = \frac{\sigma\beta_1\beta_2(\lambda+\alpha)\left(1-e^{-(\lambda/\mu)}\right)}{\lambda M_2} \tag{52}$$

$$P^*_{0,0}(0) = \frac{\sigma\beta_1\beta_2\left[(\lambda+\alpha)\left(e^{-(\lambda/\mu)}+\frac{\lambda}{\mu}-1\right)+\frac{\lambda^2}{\mu}\{p(1-q)+q\}e^{-(\lambda/\mu)}\right]}{\lambda M_2} \tag{53}$$

$$Q_{0,1} = \frac{\lambda(1-q)\beta_1\beta_2 e^{-(\lambda/\mu)}}{M_2} \tag{54}$$

$$R_O = \frac{\lambda\sigma\beta_1(1-q)(1-c)e^{-(\lambda/\mu)}}{M_2} \tag{55}$$

$$R_S = \frac{\alpha\sigma\beta_2(1-c)e^{-(\lambda/\mu)}}{M_2} \tag{56}$$

$$A_V = \frac{\sigma\beta_1\beta_2\left\{\lambda+\alpha\left(1-e^{-(\lambda/\mu)}\right)\right\}}{\lambda M_2} \tag{57}$$

5. Numerical findings:

This section has dealt extensively with the numerical distribution of the availability of the one-operating system with one standby component. We set the values of the parameters for numerical simulation as follows [Ref. 9]:

$\lambda = 0.6$, $\mu = 20$, $\beta_1 = 60$, $\beta_2 = 75$, $\alpha = 0.5$, $\sigma = 50$, $q = 0.6$, $c = 0.8$, $p = 0.9$, $b_1 = 0.04$

We analyze three repair time distributions such as Exponential (M), n – stage Erlang (Er_n) and deterministic (D) for comparative and demonstration purposes.

We explore following cases for numerical computing and examine the influence of various parameters $\lambda, \alpha, \mu, \beta_1, \beta_2$, c, p, q and σ on the availability and probabilities of three repair time distributions shown in Tables 1 – 5.

Case 1: Availability and state probabilities of the system.

Distribution	M	Er_n	D
Parameters	$\mu = 20$	$\mu = 20$	$\mu = 20$
$P_{1,1}$	0.93875248	0.93824823	0.93791568
$P_{1,0}$	0.06255786	0.06348531	0.06375726
$P_{0,0}$	0.05975843	0.05509873	0.05467325
$Q_{0,1}$	0.00654638	0.00652579	0.00652315
R_O	0.00007698	0.00007698	0.00007698
R_S	0.00005368	0.00005368	0.00005368
A_V	0.98643548	0.98724987	0.98784652

Table 1

Case 2: The value of μ vary and other parameters remain constant. After that we see effect

Indices	Distribution	μ								
		20	22	24	26	28	30	32	34	36
$P_{1,1}$	M	0.9436	0.9438	0.9439	0.9440	0.9441	0.9442	0.9443	0.9444	0.9445
	Er_n	0.9436	0.9437	0.9439	0.9439	0.9441	0.9441	0.9443	0.9444	0.9445
	D	0.9436	0.9437	0.9439	0.9439	0.9441	0.9441	0.9443	0.9444	0.9445
A_V	M	0.9828	0.9835	0.9838	0.9844	0.9851	0.9855	0.9858	0.9859	0.9865
	Er_n	0.9845	0.9847	0.9848	0.9849	0.9852	0.9858	0.9863	0.9868	0.9874
	D	0.9852	0.9855	0.9857	0.9858	0.9859	0.9860	0.9865	0.9875	0.9888

on availability and state probability $P_{1,1}$.

Table 2

Case 3: The value of λ vary and other parameters remain constant. After that we see effect on availability and state probability $P_{1,1}$.

Indices	Distribution	λ								
		0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$P_{1,1}$	M	0.9683	0.9605	0.9579	0.9540	0.9535	0.9498	0.9454	0.9418	0.9385
	Er_n	0.9683	0.9605	0.9573	0.9539	0.9535	0.9498	0.9449	0.9418	0.9382
	D	0.9683	0.9605	0.9573	0.9539	0.9535	0.9498	0.9449	0.9418	0.9382
A_V	M	0.9949	0.9935	0.9932	0.9925	0.9905	0.9873	0.9858	0.9846	0.9819
	Er_n	0.9958	0.9944	0.9939	0.9935	0.9918	0.9878	0.9873	0.9868	0.9854
	D	0.9972	0.9975	0.9965	0.9953	0.9948	0.9886	0.9875	0.9872	0.9865

Table 3

Case 4: The value of α vary and other parameters remain constant. After that we see effect on availability and state probability $P_{1,1}$.

Indices	Distribution	α								
		0.010	0.015	0.020	0.025	0.030	0.035	0.040	0.045	0.050
$P_{1,1}$	M	0.9253	0.9252	0.9251	0.9250	0.9249	0.9248	0.9247	0.9247	0.9246
	Er_n	0.9253	0.9251	0.9251	0.9249	0.9249	0.9247	0.9246	0.9247	0.9245
	D	0.9253	0.9251	0.9251	0.9249	0.9249	0.9247	0.9246	0.9247	0.9245
A_V	M	0.9675	0.9669	0.9665	0.9662	0.9659	0.9655	0.9654	0.9652	0.9642
	Er_n	0.9682	0.9671	0.9668	0.9665	0.9662	0.9658	0.9658	0.9655	0.9654
	D	0.9691	0.9685	0.9674	0.9672	0.9668	0.9665	0.9663	0.9658	0.9656

Table 4

Case 5: The value of σ vary and other parameters remain constant. After that we see effect on availability and state probability $P_{1,1}$.

Indices	Distribution	σ								
		20	25	30	35	40	45	50	55	60
$P_{1,1}$	M	0.9132	0.9145	0.9152	0.9153	0.9156	0.9161	0.9168	0.9173	0.9178
	Er_n	0.9132	0.9142	0.9152	0.9153	0.9154	0.9161	0.9165	0.9169	0.9178
	D	0.9132	0.9142	0.9152	0.9153	0.9154	0.9161	0.9165	0.9169	0.9178
A_V	M	0.9465	0.9469	0.9471	0.9475	0.9478	0.9478	0.9479	0.9484	0.9489
	Er_n	0.9469	0.9472	0.9473	0.9476	0.9481	0.9482	0.9483	0.9488	0.9492
	D	0.9475	0.9478	0.9479	0.9482	0.9487	0.9489	0.9491	0.9495	0.9498

Table 5

6. Conclusion:

The availability of a warm standby system is shown in this study. This system is made up of one primary and one standby unit that deal with two forms of reboot delay and switching failure. When the primary unit fails, it is immediately replaced by a standby unit. The failed units are repaired on a first come, first serve basis. First, we calculate the steady-state availability of this system using the supplementary variable technique and the Laplace transformation. We examine numerical findings based on explicit representations of availability and state probability for three repair time distributions, namely; exponential, n - stage Erlang, and deterministic distributions. On the basis of this computation we find that deterministic distribution makes better result than other two distributions exponential and n - stage Erlang respectively.

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